Waiting and loss probabilities for route nodes

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Abstract

One major aim of railway operations research is to identify bottlenecks in the infrastructure. In the past, lines were analysed in detail but junctions were often neglected because of their complexity. The consequences become evident at the "ends" of various European high-speed projects. For the purpose of assessing capacity, junctions can be split into route nodes and sets of station tracks. Route nodes contain the switching zones in the throats of stations, linking the adjacent lines and/or station tracks. They allow several train moves to be performed simultaneously as long as these do not conflict in whole or part.

Besides simulation approaches, recourse is also had when assessing the capacity of railway infrastructure to analytical methods based on queueing theory. These analytical methods establish a correlation between the level of utilisation of infrastructure and resultant performance parameters such as waiting times or waiting probabilities.

Analytical methods can even be adopted without access to a specific timetable. It is sufficient to know the quantities of each different type of train involved, the train mix being calculated stochastically. Calculations take account of the minimum headway times between train moves. Analytical models are used for long-term or strategic network planning. There is usually no detailed timetable available for such long planning horizons, just a certain amount of general information on the intended transport schedule. Another advantage of analytical approaches is that computing times are fast.

Analytical methods of calculating the capacity of lines and station tracks are already very widespread and have been incorporated into a number of software tools. Approximative solutions have provided the sole means of assessing route nodes hitherto, however.

This paper describes an algorithm for calculating the waiting and loss probabilities for a route node. The approach adopted uses a multi-resource queue to model a route node, an area of track over which two or even more train moves can, after all, be performed simultaneously. First, the system's basic characteristics are described. An equation for calculating the exact loss probability of the system is then extrapolated before conducting an approximation exercise to deduce waiting probabilities. The system's capacity is arrived at by comparing the waiting probabilities calculated with an acceptable "level of service".

Keywords:

route node, capacity assessment, queueing theory

1 Introduction

Economical running of the railway system is predicated upon track facilities being correctly dimensioned. The high level of investment, long planning periods and long service lives associated with such track facilities mean that any change in the way they impact on performance capacity as well as in their own performance patterns needs to be studied and assessed. The performance capacity of track facilities depends not only on the infrastructure available but also on the loading to which they are subjected in the form of the transport schedule being put to effect and the quality of conveyance and transportation underpinning it.

Dimensioning railway infrastructure generally involves dividing it up into lines and nodes. Whereas a number of procedures for calculating the performance capacity of lines have established themselves in recent years [15], [2], only tentative methods of determining the capacity of nodes are as yet available [17], despite the fact that railway nodes are often where congestion actually occurs [19]. The primary reason for this is that nodes are considerably more complex in nature than lines and are more difficult to model. As well as comprising a set of station tracks, for instance, a node also includes the switching zones in station throats, known as route nodes. Route nodes link the adjoining lines with the set of station tracks. Figure 1 shows in schematic form the division of a node into route nodes and a set of station tracks.

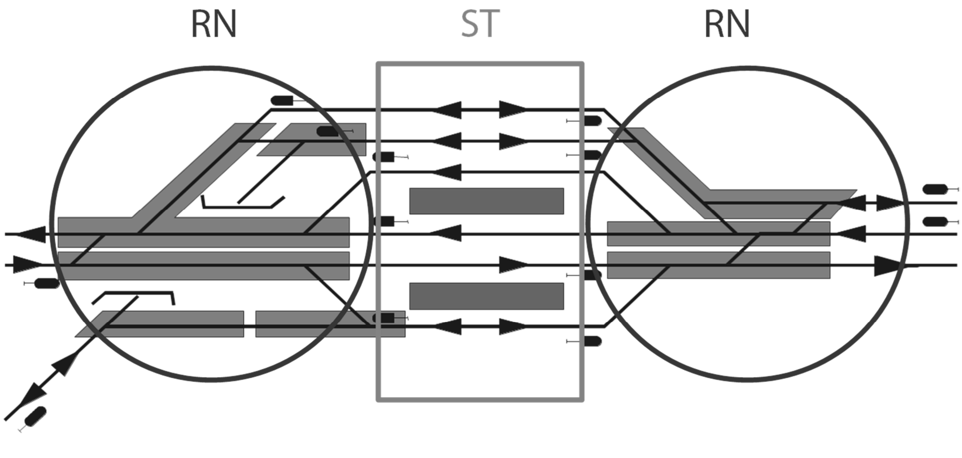

Figure 1: Modelling railway nodes with route nodes (RN) and a set of station tracks (ST)

The present paper sets out a method of determining the performance capacity of route nodes. Progress to date is detailed in Section 2, in which attention is also given to a variety of methods of determining node capacity. Sections 3 and 4 draw on [10] and describe the modelling of a route node as a multi-resource queue. An equation for the exact computation of loss probabilities in route nodes is extrapolated. As a means of determining performance capacity, the following Section 5 sets out a method whereby waiting probabilities can be approximatively determined. By way of conclusion, the equations elucidated are applied in a sample computation in Section 6.

2 Progress to Date

Determining the capacity of railway facilities constitutes a key challenge for railway operations research. Further factors impacting on capacity besides the infrastructure being dimensioned are the transport schedule under review and the accepted level of quality. A track facility's capacity is essentially the number of service enquiries that can be processed to an accepted level of quality within the period under review.

This loading-based level of quality can be defined in a number of ways. The first involves a set of parameters used to rate the timetable that are solely dependent on the rate of utilisation, one example being occupation ratio ρ. Further quality requirements serve to rate traffic quality on the basis, for instance, of levels of punctuality. Use is also made in railway operations research of waiting times or waiting probabilities as levels of quality, to conclude. It is possible to establish waiting times and waiting probabilities both for the compilation of timetables (scheduled waiting times) and for the operation of trains (unscheduled waiting times) [18]. All of the parameters referred to reveal a correlation between quality and a track facility's rate of utilisation: any increase in the rate of utilisation induces changes in the quality indicator.

A track facility's theoretical performance capacity, its performance limit, is attained once the system continuously receives more service enquiries than it can deal with. This is a scenario that would give rise to an infinite queue. Theoretical performance capacity is merely a parameter with which to describe the system and is not suitable for practical capacity quantification exercises. Reference is made to [11] where determining the performance limit for route nodes is concerned. Figure 2 illustrates the correlation between a track facility's rate of utilisation and its 'occupation ratio' and 'traffic quality' parameters based on levels of punctuality and waiting times.

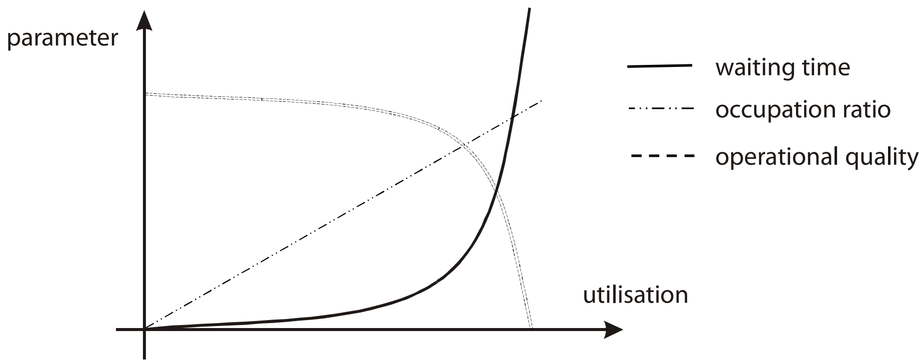

Figure 2: Correlation between rate-of-utilisation and quality parameters

It is the practical potential for performance that is of interest when establishing capacity. This is arrived at by stipulating a specific level of service, an operation that involves defining either an admissible occupation ratio as the implicit level of service or an admissible waiting time as the explicit level of service. It is then possible to determine the admissible rate of utilisation for this stipulated level of quality by computing the rate of utilisation that would be achieved at this level of service.

Figure 3 portrays the correlation between a track facility's rate of utilisation and the ensuing waiting times. If the level of service is specified as being an admissible waiting time, it can be concluded that the facility's capacity is the optimum number of trains n_{opt}. It is thus plain to see that capacity is a variable entity. It is also technically feasible to process more than the optimum number of trains, though this is likely to lead to a drop in quality.

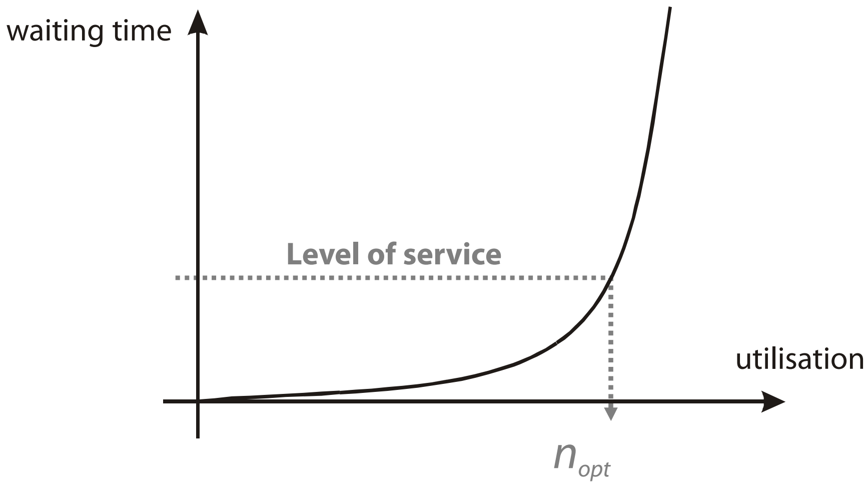

Figure 3: Determining the optimum number of trains for a predefined level of service

The two following sections contain an elucidation of the methods adopted to determine node capacity (Subsection 2.1) as well as a discussion of the "levels of service" employed as rating benchmarks (Subsection 2.2).

2.1 Methods of determining node capacity

The multifarious interactions between, and wider network impact of, railway nodes mean they are often complex structures that are awkward to analyse. It is usual, therefore, to divide a node up into a set of station tracks and the accompanying route nodes. Route nodes are notably distinguished by the fact that, unlike on open line track, it is sometimes possible - depending on how the routes are set - for several train moves to negotiate them in tandem. There are a variety of means of rating and quantifying the capacity of nodes in railway operations research.

One method involves recreating traffic patterns at a railway node in as realistic a manner as possible. Tools that allow the operation of trains to be simulated are a suitable means of achieving this. Forming their point of departure are a microscopic infrastructure model and the train moves in an archived timetable. In the course of conducting a large number of Monte-Carlo simulation runs, disruptions are introduced into the timetable that may lead to (knock-on) conflicts, and altered running slots as a result, during the simulation exercise. When evaluating such multiple simulation runs, attention focuses on deviations from the original timetable. These generally concern operating parameters such as levels of punctuality or increases/decreases in delays. One microscopic simulation tool currently in use, for instance, is the LUKS® method [6].

Alongside generically generated simulation data, use can also be made of actual operating data for analysis and evaluation purposes. This approach is particularly suitable when evaluating the stability of the actual timetable. Diverse means of evaluating and analysing operating data are detailed in [5].

Another potential procedure is to consider capacity with the aid of "compilatory methods", i.e. by determining the capacity consumed under the timetable. The method most commonly adopted involves compressing a timetable pursuant to UIC Code 406 [15]. UIC Code 406 draws on the concatenation method devised by ADLER [1] in which timetabled train-paths (stepped blocking-time series) are pushed as close together as possible. UIC Code 406 addresses itself to the concatenation of sections of line; extensions of its scope to embrace the calculation of node capacity are the subject of ongoing discussion in the relevant specialist literature [8], [9] and are currently being elaborated by the UIC [17].

An admissible occupation ratio is used as the quality benchmark under the concatenation method. Guideline figures arrived at with reference to selected illustrative lines adopting the analytical methods set out below are given in UIC Code 406 [20].

Analytical methods of establishing capacity also exist, furthermore. Under these, an equation-style correlation is established between the track facility's rate of utilisation and the ensuing parameter - primarily waiting probabilities/waiting times. These methods draw on the queueing theory, under which customer demands are processed in single or multiple-channel queueing systems. More detailed information on the queueing theory for railways is contained in [18]. One particular advantage of the analytical approach is that no actual timetable is required as an input variable as with other methods, general information on the transport schedule in the form of the train mix being sufficient. Analytical methods are most notably adopted in strategic network planning where there is as yet no timetable with which to assess future capacity quantification requirements. The computing times involved with analytical methods are far shorter than for simulation exercises, moreover.

A method originated by HERTEL [7] exists for quantifying the capacity of station tracks in railway nodes by analytical means. Approximative approaches devised by POTTHOFF [13] and OETTING [12] are available for route nodes. An exact specification of loss probabilities for route nodes is set out in [10]. The following sections of the present paper set out how waiting probabilities for a route node can be determined on the basis of loss probabilities.

2.2 Quality benchmark as "level of service"

The methods now adopted for capacity studies operate with various types of quality benchmark. Simulation exercises or service-data evaluations are performed to analyse parameters such as punctuality at selected cross-sections or increases/decreases in delays. There is no uniform application or directive for defining such quality benchmarks.

The method specified in UIC Code 406 merely gauges a track facility's rate of utilisation on the basis of its occupation ratio. The timetable structure is factored in but no consideration is given to features such as the rankings (priorities) of train moves or to delay parameters. UIC Code 406 enumerates uniform limit values for defining a quality benchmark on the basis of admissible levels of occupation [15].

Analytical methods can be adopted to establish both scheduled and unscheduled waiting times and probabilities. Use is specifically made of the admissible quality parameters in Germany and they are incorporated into directives issued by DB Netz AG for this reason [3].

All the methods mentioned give sole consideration to train running factors, whereas more recent research work additionally addresses economic aspects. To this end the outgoings and earnings of railway infrastructure managers and train operating companies are calculated and compared as a function of a given track facility's rate of utilisation. The optimum range from an economic point of view is deemed to be that within which a rate of utilisation yielding the greatest possible profit (or lowest possible loss) is achieved. Reference is made to [14] for more detailed information on economically optimum numbers of trains.

3 Modelling a Route Node as a Multi-Resource Queue

This Section contains a formal description of how a route node is modelled, and how differing train moves (customer types) are served, for subsequent computation. A route node is made up of a total of s queueing channels. There can be no more than one customer in any one queueing channel. A queueing channel thus corresponds to a sectional route node for the purposes of railway operations research [16]. Figure 4 illustrates the queueing channels (sectional route nodes) into which a route node at a station throat is divided. In the example shown, the route node to the left of the set of station tracks is divided into seven queueing channels, which are colour-highlighted.

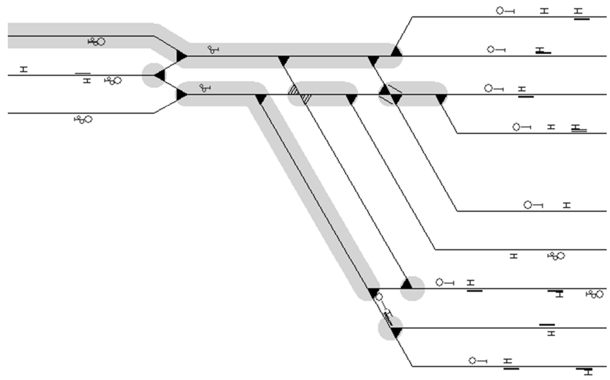

Figure 4: Queueing channels in a route node

The queueing channels are numbered consecutively as $r = 1 \ldots s$. The quantity of queueing channels is taken as being R. The capacity vector $\boldsymbol{c}$ of the system is

$$\boldsymbol{c} = \left(c_1, c_2, \ldots, c_r, \ldots, c_s\right) \tag{1}$$

$$\text{where } c_r = 1 \, \forall \, r \, . \tag{2}$$

There are a total of g different customer types occupying one or more channels in the route node. Customer types are designated as $j = 1 \ldots g$ and differ owing to their differing channel requirements. Let the arrival rate of customers of customer type j be λ_j. This is obtained by dividing the number of incoming customers n_j of customer type j by period of time t_U.

$$\lambda_j = \frac{n_j}{t_U} \tag{3}$$

Let the total number of all customers arriving in period of time t_U be n_{tot}.

$$n_{tot} = \sum_{j=1}^{g} n_j \qquad (4)$$

It is assumed that the intermediate arrival times of a stream of demands are statistically independent and of identical distribution. The overall arrival rate of all customers λ is the total number of all customers divided by the period under review. It can alternatively be established using the sum of the arrival rates of the individual customers.

$$\lambda = \frac{n_{tot}}{t_U} = \sum_{j=1}^{g} \lambda_j \qquad (5)$$

Occupation matrix Δ denotes channels occupied by a customer. The occupation matrix is a Boolean matrix measuring g by s.

$$\mathbf{\Delta} = \left(\delta_{jr}\right) \qquad (6)$$

$$\delta_{jr} = \begin{cases} 1 & \text{if customer } j \text{ occupies channel } r, \\ 0 & \text{otherwise.} \end{cases} \qquad (7)$$

Row j in the occupation matrix details requirement $\boldsymbol{\delta}_{j\bullet}$ of customer j.

$$\boldsymbol{\delta}_{j\bullet} = \left(\delta_{j1}, \delta_{j2}, ..., \delta_{jr}, ..., \delta_{js}\right) \qquad (8)$$

Any instance of occupation by a customer begins simultaneously for all channels required. Once customer j has been served at service rate μ_j, all occupied channels are re-cleared at the same time. This means any route that has been set is completely cancelled. It is assumed that service times are statistically independent of one another and identically distributed.
Conflict matrix $\boldsymbol{A}$ is a Boolean matrix measuring g by g. It denotes whether two customers can be served simultaneously or whether they give rise to a conflict in the route node. Let

$$\boldsymbol{A} = \left(a_{ij}\right) \qquad (9)$$

where

$$a_{ij} = \begin{cases} 1 & \text{if customers } i \text{ and } j \text{ occupy at least one channel together,} \\ 0 & \text{otherwise.} \end{cases} \qquad (10)$$

The conflict matrix can be calculated for the customers' requirements $\boldsymbol{\delta}_{j\bullet}$ as follows:

$$a_{ij} = \min\left\{\boldsymbol{\delta}_{i\bullet} \cdot \boldsymbol{\delta}_{j\bullet}^{\mathrm{T}}, 1\right\}. \tag{11}$$

The service rate of a customer μ_j is the inverse of the latter's mean service time. The occupation ratio ρ_j of a customer j is defined as being the ratio of the latter's arrival and service rate.

$$\rho_j = \frac{\lambda_j}{\mu_j} \tag{12}$$

At any given time the system is either empty or else at least one customer is being served. Whether several customers can be served at the same time depends on their requirements $\boldsymbol{\delta}_{j\bullet}$. It is possible with the aid of customer combinations $\boldsymbol{k}$ to describe the system's statuses. A customer combination is possible if all customers forming part of the combination can be served at the same time. Let the number of all possible customer combinations with at least one customer be m. The trivial scenario in which there is no customer in the system is defined as $\boldsymbol{k}_0$. Combinations are numbered consecutively as $l = 0...m$. Combination $\boldsymbol{k}_l$ states which customers occur in combination l. A combination is described in the form of

$$\boldsymbol{k}_l = \left(k_{1l}, k_{2l}, ..., k_{jl}, ..., k_{gl}\right) \tag{13}$$

where

$$k_{jl} = \begin{cases} 1 & \text{if combination } l \text{ involves one customer,} \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The trivial combination arises when

$$k_{j0} = 0 \;\forall\; j = 1...g. \tag{15}$$

It holds that combination $\boldsymbol{k}_l$ is a possible combination at the precise moment when

$$\boldsymbol{k}_l \cdot \boldsymbol{\Delta} \leq \boldsymbol{c}\,. \tag{16}$$

Let the number of all possible combinations be Ψ:

$$\Psi = \left(\boldsymbol{k}_0, \boldsymbol{k}_1, \boldsymbol{k}_2, ..., \boldsymbol{k}_l, ..., \boldsymbol{k}_m\right) \tag{17}$$

Combination $\boldsymbol{k}_l$ may occupy one or more channels in the system. Combined occupation matrix $\boldsymbol{\Gamma}$ denotes the channels occupied given combinations $\boldsymbol{k}_l$. $\boldsymbol{\Gamma}$ is an m-by-s matrix.

$$\boldsymbol{\Gamma} = \left(\gamma_{lr}\right) \tag{18}$$

$$\gamma_{lr} = \begin{cases} 1 & \text{if channel } r \text{ is occupied given combination } l, \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

$$\gamma_{lr} = \sum_{j=1}^{g} k_{jl} \cdot \delta_{jr} \tag{20}$$

Quantity Ψ is now allocated as a function of an incoming customer j. Let quantity $\Omega_j \subset \Psi$ be the quantity of combinations in which an incoming customer j can immediately be served; i.e. assuming that all channels required by customer j are available. The elements in Ω_j can be characterised as follows:

$$\boldsymbol{k}_l \in \Omega_j \Leftrightarrow \delta_{jr} + \gamma_{lr} \leq 1 \quad \text{where } r = 1 \ldots s. \tag{21}$$

Let the complementary quantity for Ω_j be Φ_j, i.e.

$$\Phi_j \subset \Psi \tag{22}$$

and

$$\Omega_j \cup \Phi_j = \Psi\,. \tag{23}$$

Quantity Φ_j contains all combinations in which an incoming customer j cannot be served owing to at least one required channel being occupied. It follows from the characterisation of Φ_j that

$$\boldsymbol{k}_l \in \Phi_j \Leftrightarrow \delta_{jr} + \gamma_{lr} > 1 \text{ for at least one channel } r. \tag{24}$$

The system's stationary status Π can be described in terms of status probabilities. There is a certain probability π_l that the system will be in the status of combination l. The probability of there being no customer in the system is denoted by means of π_0.

$$\Pi = \{\pi_0, \pi_1, \pi_2, \ldots, \pi_l, \ldots, \pi_m\} \tag{25}$$

In respect of status probabilities, it holds that

$$0 \leq \pi_l \leq 1 \tag{26}$$

and

$$\sum_{l=0}^{m} \pi_l = 1\,. \tag{27}$$

4 Calculating Loss Probabilities in a Route Node

DZIONG and ROBERTS [4] set forth an algorithm that enables the loss probabilities for a multi-resource queue to be determined.
Establishing loss probabilities p_V first involves determining status probabilities π_l for the stationary status. It holds for each combination $\boldsymbol{k}_l$ that:

$$\pi_l = \pi_0 \cdot \prod_{j=1}^{g} \rho_j^{k_{jl}} \tag{28}$$

In the following operation on the normalising condition (27)

$$\sum_{l=0}^{m} \pi_l = 1 ,$$

first the initial addend is written out

$$\pi_0 + \sum_{l=1}^{m} \pi_l = 1 \tag{29}$$

and then equation (28) is inserted

$$\pi_0 + \sum_{l=1}^{m} \left(\pi_0 \cdot \prod_{j=1}^{g} \rho_j^{k_{jl}} \right) = 1 . \tag{30}$$

A process of conversion yields

$$\pi_0 = \frac{1}{1 + \sum_{l=1}^{m} \prod_{j=1}^{g} \rho_j^{k_{jl}}} . \tag{31}$$

Since it holds for combinations k_{j0} that

$$\prod_{j=1}^{g} \rho_j^{k_{j0}} = 1 , \tag{32}$$

it is possible to reduce equation (31) to

$$\pi_0 = \frac{1}{\sum_{l=0}^{m}\prod_{j=1}^{g}\rho_j^{\,k_{jl}}}\,. \tag{33}$$

The inverse of π_0 can be regarded as constituting normalisation constant G.

$$\mathrm{G} = \pi_0^{-1} = \sum_{l=0}^{m}\prod_{j=1}^{g}\rho_j^{\,k_{jl}} \tag{34}$$

With status π_0 and the normalisation constant having been defined, it is now possible to compute all other statuses with the aid of (28).

$$\pi_l = \frac{\prod_{j=1}^{g}\rho_j^{\,k_{jl}}}{\mathrm{G}} \tag{35}$$

The loss probability $p_{V,j}$ of a customer *j* equals the sum of status probabilities π_l in which an incoming customer *j* is not admitted.

$$p_{v,j} = \sum_{l \text{ where } \boldsymbol{k}_l \in \Phi_j} \pi_l \tag{36}$$

The system's complexity increases rapidly. This is due to the customer combination options, which increase very rapidly in the case of major nodes. A method is accordingly set out in [10] whereby the degree of complexity can be lessened without any loss in computing accuracy by breaking the overall system up into several subsystems.

5 Determining Waiting Probabilities in a Route Node

Railway operations research makes no use of the loss probabilities calculated in the previous section, since it generally has queueing systems as opposed to loss systems at its disposal. It is therefore necessary to extrapolate waiting probabilities from the loss probabilities calculated. POTTHOFF [13] uses the loss probabilities calculated to determine the waiting probabilities for a set of station tracks. It is fundamentally the case that loss and waiting probabilities are virtually identical for low levels of occupation and only begin to diverge significantly given higher levels (cf. Figure 7).

An approximative solution is adumbrated below that establishes the waiting probabilities in a route node on the basis of the applicable loss probabilities. To this end, consideration is first given to the loss and waiting probabilities for a single-channel queueing system involving random arrival and service times. It holds for loss probability $p_{v,1}$ and waiting probability $p_{w,1}$ under such a system that:

$$p_{v,1} = \frac{\lambda}{\lambda + \mu} \tag{37}$$

$$p_{w,1} = \frac{\lambda}{\mu} \tag{38}$$

Correlating waiting and loss probabilities yields a loading-dependent factor *f*:

$$f = \frac{p_{w,1}}{p_{v,1}} = \frac{\lambda + \mu}{\mu} = 1 + \rho \tag{39}$$

If this factor is now also applied to the route node, the following is arrived at as an approximative solution for determining waiting times in a route node:

$$p_{w,j} = (1 + \rho_j) \cdot p_{v,j} \tag{40}$$

The loss probabilities for customer type *j* are accorded a supplementary factor through this process of approximation. The quality of approximation depends on the number of further customer types involved. The quality of this approximative solution diminishes with each further customer interacting with customer type *j* - i.e. conflicting with it.

6 Sample Computation

Let use be made of the route node comprising 7 queueing channels shown in Figure 4. The queueing channels are numbered as follows.

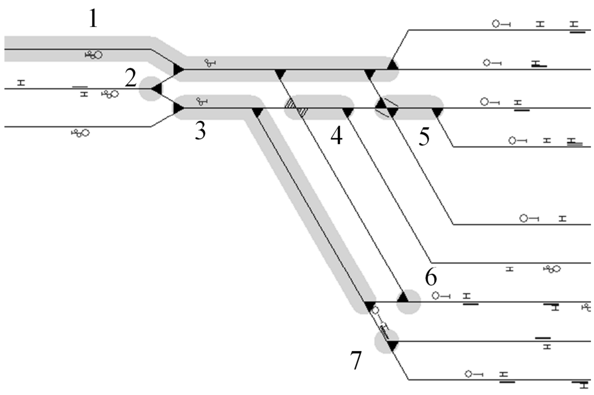

Figure 5: Queueing channels one to seven in a route node

A total of five different customer types avail themselves of this queueing system, as illustrated in the following Figure.

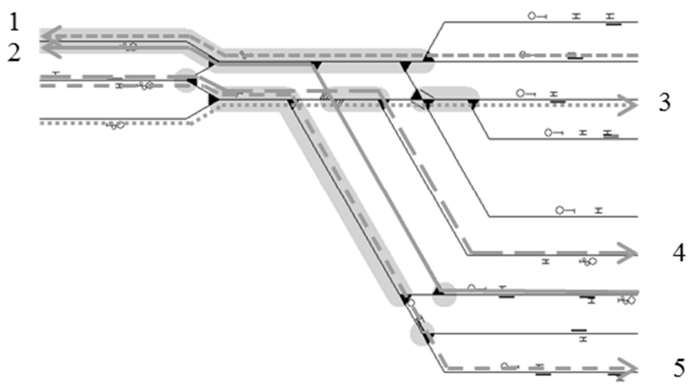

Figure 6: Customer requirements for a route node

Occupation matrix $\mathbf{\Delta}$ for this queueing system is thus as follows:

$$\mathbf{\Delta} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Let the arrival and service rates for customers be:

Cust. type	**Arrival rate λ**	**Service rate μ**
1	0.06	0.5
2	0.02	0.4
3	0.03	0.6
4	0.04	0.5
5	0.05	0.3

Table 1: Arrival rate λ and service rate μ by customer type

The overall arrival rate λ in this example is

$$\lambda = \sum_{j=1}^{g} \lambda_j = 0.20\,.$$

This illustrative example yields the following loss and waiting probabilities for the five customer types.

Cust. type	**Loss probability p_v**	**Waiting probability p_w**
1	0.1416	0.1586
2	0.2277	0.2391
3	0.2586	0.2715
4	0.2586	0.2793
5	0.2255	0.2631

Table 2: Calculation of loss and waiting probabilities

The average values computed across all customers yield a loss probability of 21.21 % and a waiting probability of 23.38 %. The loss and waiting probabilities for other rates of utilisation are likewise calculated by varying the arrival rates whilst retaining the same mixing ratio. In addition, the waiting probabilities established are compared with the values from a Monte-Carlo simulation exercise. The results are shown in the following Figure.

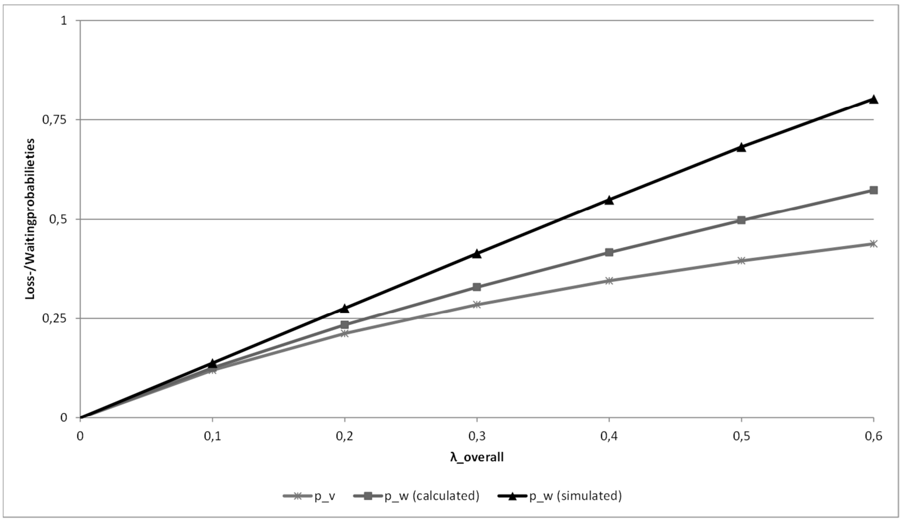

Figure 7: Loss and waiting probabilities for various arrival rates

At low rates of utilisation roughly up to an overall arrival rate λ of 0.2, there is little to separate loss and waiting probabilities. The difference becomes more pronounced where loadings are greater.

7 Concluding Summary and Outlook

Use is made of a variety of rail service research methods when determining node capacity these days. An existing timetable is evaluated under the compilatory method, whilst simulation and traffic analyses operate with service parameters, and the analytical models used in queueing theory establish waiting times or waiting probabilities. The performance capacity of a given track facility is gauged with reference to various types of quality benchmark under these different methods.

Analytical models can only provide data on a track facility's performance capacity with reference to an anticipated future transport schedule and, for this reason and owing to the short computing times involved, are most suitable for strategic network planning.

For the purpose of establishing capacity, railway nodes are divided up into a set of station tracks and the switching zones constituting a station's "throats" (route nodes). It is possible applying the set of equations contained in this paper to exactly determine the loss probabilities for a route node. An approximative means of extrapolating waiting probabilities is pointed up. The quality of this approximative solution diminishes as the track facility's level of loading rises.

Any future refinement of analytical models is dependent upon conclusive research being conducted into enhanced means of estimating waiting probabilities, including for the heavy-traffic sphere.

There is a need, to conclude, to additionally apply internationally recognised quality benchmarks and a uniform procedure for establishing capacity to railway nodes by, for instance, extending the scope of UIC Code 406.

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